Array Shape Calibration Using Carry-on Instrumental Sensors

Wang Bu-hong, Wang Yong-liang and Chen Hui
Key Research Lab, Radar Academy, Wuhan 430010, China
E-mail: wbhcx@yahoo.com.cn

Abstract—**A novel and efficient method for calibrating a sensor array with position uncertainties is proposed in this paper. The method is based on two non-disjoint sources in unknown directions and three carry-on instrumental sensors. It can be applied to arbitrary array geometries including linear arrays. Besides, no small position error assumption is made, which is always an essential prerequisite for many existing array shape calibration techniques. The new method achieves a favorable array shape calibration just using a one-dimensional search, with no high-dimensional nonlinear search and convergence burden involved. It is also possible to extend the proposed idea to tackle the problem of direction dependent gain and phase uncertainties. Simulation results are provided to demonstrate the effectiveness and behavior of the proposed method.**

I. INTRODUCTION

Since their introduction, high-resolution direction finding algorithms such as MUSIC and ML have received significant attention. This is due to their potential ability to resolve sources separated by less than one standard beamwidth of the receiving array, unlike the conventional Fourier-based direction finding procedures. Yet despite this potential advantage offered by high-resolution methods, their application to real systems has been very limited. One of the main reasons for this situation is the practical difficulty associated with calibrating the array manifold, since the performance of these methods depends strongly on the accuracy of the array manifold. However, in practice, the actual array manifold always differs from the nominal array manifold due to sensor position errors, gain and phase perturbations, mutual coupling, etc. The presence of these unknown calibration errors is the major factor limiting the performance of the high-resolution methods in practical direction finding system. Hence to achieve high-resolution performance, array calibration is always necessary. The primary interest of this paper is focused on the sensor position errors, although the method proposed here can also be easily extended to the case of gain and phase uncertainties.

Sensor position errors induce the direction dependent sensor phase errors to the array manifold so as to exert detrimental effects on the performance of the high-resolution direction finding algorithms. Various techniques have been developed in the literature to circumvent the problem of sensor position errors. They are either active and need calibrating sources in known directions or passive and rely upon the sources present in the field to achieve self-calibration. To date, however most of these techniques do not work perfectly in the sense that they are unable to always acquire satisfactory array shape calibration due to convergence burden of multimodal nonlinear search [1]-[5], or if they can, they are either too costly to implement due to the need of auxiliary calibrating sources in known directions [6]-[8] or some certain pathological array-source geometries disable them at all [9]-[10]. Besides almost all these above array shape calibrating techniques assume that the position perturbations are relatively small deviations from the nominal positions and thus a first order approximation to the perturbed array response vector is often used to simplify the estimation procedures. However, many simulation results in [4] show that these techniques fail under even moderate perturbation errors. The motivation of this paper is to attempt to suggest an efficient and relatively inexpensive and practical scheme for the array shape calibration.

The scheme proposed here needs two non-disjoint sources in unknown directions. It relaxes the small error assumption and search convergence burden. The ambiguity problem of linear array identified in [9]-[10] can also be mitigated. The only price paid for above merits is that three carry-on instrumental sensors are needed to work as coordinate reference and at the same time introduce some more degrees of freedom to tackle the identifiability problem associated with the linear array.

II. PROBLEM FORMULATION

Consider an array of K sensors of arbitrary geometry impinged by M=2 uncorrelated sources from far field in unknown directions at $\boldsymbol{\theta}=[\theta_1,\theta_2]^T$ .The signal waveforms are assumed to be narrowband of known center frequency. The actual positions of these K sensors differ from their nominal positions. In addition, we have three carry-on instrumental sensors, whose positions are assumed precisely known and one of which is chosen as the origin of the coordinates. As a result, an array of N=K+3 sensors is formed. The complex envelope of the noise-corrupted N sensors array output vector $\boldsymbol{X}(t)$ may be written as (1):

$$\boldsymbol{X}(t)=\boldsymbol{A}(\boldsymbol{\theta})S(t)+\boldsymbol{N}(t) \quad t=1,2,\cdots L \tag{1}$$

where $\boldsymbol{S}(t)$ is a M×1 signal vector, $N(t)$ is a N×1 noise vector. It is assumed that the signals and noises are stationary, zero mean uncorrelated Gaussian random processes and further, the noises are both spatially and temporally white with variance σ^2 .Array manifold matrix $A(\boldsymbol{\theta})$ is $N\times M$ matrix whose columns are the steering vectors. In the

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
14 APR 2005	**N/A**	-

4. TITLE AND SUBTITLE
Array Shape Calibration Using Carry-on Instrumental Sensors

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Key Research Lab, Radar Academy, Wuhan 430010, China

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES
See also ADM001798, Proceedings of the International Conference on Radar (RADAR 2003) Held in Adelaide, Australia on 3-5 September 2003.

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			
unclassified	**unclassified**	**unclassified**	**UU**	**4**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

presence of sensor position uncertainties, the $N\times1$ steering vector $W(\theta_m)$ can be modeled as (2):

$$\boldsymbol{W}(\theta_m)=\boldsymbol{\Gamma}(\theta_m)\boldsymbol{a}(\theta_m) \quad m=1,2 \tag{2}$$

where $\boldsymbol{a}(\theta_m)$ is the ideal steering vector corresponding to array nominal positions and can be expressed as (3)-(4) :

$$\boldsymbol{a}(\theta_m)=\left[1, exp\left(-j\frac{2\pi}{\lambda}\tau_{m2}\right),\cdots,exp\left(-j\frac{2\pi}{\lambda}\tau_{mN}\right)\right] \quad m=1,2 \tag{3}$$

$$\tau_{mn}=[x_n,y_n][sin(\theta_m)\,cos(\theta_m)]^T \quad n=1,2,\cdots N \tag{4}$$

λ is the wavelength of the signal, $[x_n\ y_n]$ is the nominal co-ordinates of the nth sensor with respect to the reference sensor. Further, $\boldsymbol{\Gamma}(\theta_m)$ is a $N\times N$ complex diagonal matrix whose nnth entries are the angularly dependent phase distortion induced by the nth sensor position error and it can be written as (5)-(6):

$$\boldsymbol{\Gamma}(\theta_m)=\left[1, exp\left(-j\frac{2\pi}{\lambda}\Delta\tau_{m2}\right),\cdots,exp\left(-j\frac{2\pi}{\lambda}\Delta\tau_{mN}\right)\right] \quad m=1,2 \tag{5}$$

$$\Delta\tau_{mn}=[\Delta x_n,\Delta y_n][sin(\theta_m)\,cos(\theta_m)]^T \quad n=1,2,\cdots N \tag{6}$$

where $[\Delta x_n\ \Delta y_n]$ are the position disturbance associated with the nth sensor.

The array covariance matrix and its eigendecomposition are expressed as follows:

$$\boldsymbol{R}=E\left[\boldsymbol{X}(t)\boldsymbol{X}^H(t)\right]=\boldsymbol{A}\boldsymbol{R_S}\boldsymbol{A^H}+\sigma^2\boldsymbol{I} \tag{7}$$

$$\boldsymbol{R}=\sum_{i=1}^{M}\lambda_i\boldsymbol{e}_i\boldsymbol{e}_i^{\boldsymbol{H}}+\sum_{i=M+1}^{N}\lambda_i\boldsymbol{e}_i\boldsymbol{e}_i^{\boldsymbol{H}}=\boldsymbol{E_S}\boldsymbol{\Lambda_S}\boldsymbol{E_S}+\boldsymbol{E_N}\boldsymbol{\Lambda_N}\boldsymbol{E_N} \tag{8}$$

where $\{\lambda_i; i=1,2,\cdots,N; \lambda_i\geq\lambda_{i+1}\}$ and $\{\boldsymbol{e}_i; i=1,2,\cdots,N\}$ are ordered eigenvalues and corresponding eigenvectors of $\boldsymbol{R}$ respectively. The signal subspace and noise subspace of $\boldsymbol{R}$ are respectively the ranges of the matrices:

$$\boldsymbol{E_s}=[\boldsymbol{e_1},\boldsymbol{e_2},\cdots\boldsymbol{e_M}] \tag{9}$$

$$\boldsymbol{E_N}=[\boldsymbol{e_{M+1}},\boldsymbol{e_{M+2}},\cdots,\boldsymbol{e_N}] \tag{10}$$

The problem of interest here is as follows: given L array snapshots $X(t)\ t=1,2,\cdots,L$, estimate the unknown DOAs of sources, as well as the unknown sensor position uncertainties of the K sensors, $[\Delta x_n\ \Delta y_n]\ n=4,5,\cdots N\ \ (N=K+3)$.

III. ALGORITHM DESCRIPTION

Since we have 3 instrumental sensors with no sensor position uncertainties, the first 3 diagonal entries of matrix $\boldsymbol{\Gamma}(\theta)$ are all reduced to 1. The ideal steer vector $\boldsymbol{a}(\theta)$ and phase distortion matrix $\boldsymbol{\Gamma}(\theta)$ can be partitioned as follows:

$$\boldsymbol{a}(\theta)=\left[\boldsymbol{a}_1^T(\theta)\quad \boldsymbol{a}_2^T(\theta)\right]^T \tag{11}$$

$$\boldsymbol{\Gamma}(\theta)=diag\left[\boldsymbol{I}_{1\times3}\quad [vecd(\boldsymbol{\Gamma}_2)]^T\right] \tag{12}$$

where the 3×1 vector $\boldsymbol{a}_1(\theta)$ and $K\times1$ vector $\boldsymbol{a}_2(\theta)$ are formed from the part of the elements of $a(\theta)$ corresponding to the instrumental sensors and the position-disturbed sensors respectively. Similarly, the diagonal entries of $K\times K$ diagonal matrix $\boldsymbol{\Gamma}_2(\theta)$ consist of the unknown direction dependent phase uncertainties induced by the corresponding position-disturbed sensors. Then the steer vector $\boldsymbol{W}(\theta)$ can be reformulated as follows:

$$\begin{aligned}\boldsymbol{W}(\theta)&=\boldsymbol{\Gamma}(\theta)\boldsymbol{a}(\theta)\\&=\begin{bmatrix}\boldsymbol{a}_1(\theta) & \boldsymbol{0}_{3\times K}\\ \boldsymbol{0}_{K\times1} & diag[\boldsymbol{a}_2(\theta)]\end{bmatrix}\begin{bmatrix}1\\ vecd(\boldsymbol{\Gamma}_2(\theta))\end{bmatrix}\\&=\tilde{\boldsymbol{\alpha}}(\theta)\boldsymbol{\delta}(\theta)\end{aligned} \tag{13}$$

where $\tilde{\boldsymbol{\alpha}}(\theta)$ is a $N\times(K+1)$ matrix while $\boldsymbol{\delta}(\theta)$ is a $(K+1)\times1$ vector. $diag[\boldsymbol{v}]$ denotes a diagonal matrix whose diagonal entries is formed from the elements of vector v and $vecd[\boldsymbol{A}]$ denotes a column vector where the diagonal elements of $\boldsymbol{A}$ form the vector.

The underlying basis for subspace-based DOA estimation algorithms is the orthogonality between the noise subspace and signal subspace of array covariance matrix $\boldsymbol{R}$, which means that

$$\boldsymbol{W}^H(\theta)\boldsymbol{E_N}\boldsymbol{E_N^H}\boldsymbol{W}(\theta)=0 \tag{14}$$

Substitution of (13) into (14) yields (15-17):

$$\boldsymbol{\delta}^H(\theta)\tilde{\boldsymbol{\alpha}}^H(\theta)\boldsymbol{E_N}\boldsymbol{E_N^H}\tilde{\boldsymbol{\alpha}}(\theta)\boldsymbol{\delta}(\theta)=0 \tag{15}$$

$$\boldsymbol{\delta}^H(\theta)\boldsymbol{Q}(\theta)\boldsymbol{\delta}(\theta)=0 \tag{16}$$

$$\boldsymbol{Q}(\theta)=\tilde{\boldsymbol{\alpha}}^H(\theta)\boldsymbol{E_N}\boldsymbol{E_N^H}\tilde{\boldsymbol{\alpha}}(\theta) \tag{17}$$

where $\boldsymbol{Q}(\theta)$ is a $(K+1)\times(K+1)$ Hermitian matrix. Since the $\boldsymbol{\delta}(\theta)\neq\boldsymbol{0}$, (16) means that the matrix $\boldsymbol{Q}(\theta)$ is singular, i.e., $rank[Q(\theta)]<K+1$. Note that under the condition that array manifold $\{\boldsymbol{W}(\theta): -\pi/2\leq\theta\leq\pi/2\}$ is unambiguous, the matrix $\boldsymbol{Q}(\theta)$ is singular or rank reduction if and only if the $\theta=\theta_i\ i=1,2$, since the dimension of signal subspace of $\boldsymbol{R}$ is 2. Based on this idea, we develop a DOA estimator as (18) or (19) and a array shape calibration algorithm as (20)-(26):

$$\hat{\theta}=\arg\max_{\theta}\frac{1}{\lambda_{\min}\left[\hat{\boldsymbol{Q}}(\theta)\right]} \tag{18}$$

$$\hat{\theta}=\arg\max_{\theta}\frac{1}{\det\left[\hat{\boldsymbol{Q}}(\theta)\right]} \tag{19}$$

$$\hat{\boldsymbol{\delta}}(\hat{\theta})=\boldsymbol{e}_{\min}\left[\hat{\boldsymbol{Q}}(\hat{\theta})\right] \quad with \quad \boldsymbol{e}_{\min}(1)=1 \tag{20}$$

$$\hat{\boldsymbol{Q}}(\theta)=\tilde{\boldsymbol{\alpha}}^H(\theta)\hat{\boldsymbol{E}}_{\boldsymbol{N}}\hat{\boldsymbol{E}}_{\boldsymbol{N}}^{\boldsymbol{H}}\tilde{\boldsymbol{\alpha}}(\theta) \tag{21}$$

$$vecd\left(\hat{\Gamma}_2(\hat{\theta})\right)=\left[\hat{\delta}(2)\hat{\delta}(3),\cdots,\hat{\delta}(K+1)\right]^T \tag{22}$$

$$[\Delta X\ \Delta Y]=\left[P(\hat{\theta}_1)\,P(\hat{\theta}_2)\right]\begin{bmatrix}\sin\theta_1 & \sin\theta_2\\ \cos\theta_1 & \cos\theta_2\end{bmatrix}^{-1} \tag{23}$$

$$\Delta X=[\Delta x_4\ \Delta x_5\cdots\Delta x_N]^T \tag{24}$$

$$\Delta Y=[\Delta y_4\ \Delta y_5\cdots\Delta y_N]^T \tag{25}$$

$$P(\theta_m)=-\frac{\lambda}{2\pi}angle\left[vecd(\hat{\Gamma}_2(\hat{\theta}_m)\right] \quad m=1,2 \tag{26}$$

where $\hat{\boldsymbol{E}}_N$ denotes the finite sample estimate of noise subspace $\boldsymbol{E_N}$. $\lambda_{\min}\left[\hat{\boldsymbol{Q}}(\theta)\right]$ is the smallest eigenvalue of the matrix $\hat{\boldsymbol{Q}}(\theta)$, $\boldsymbol{e}_{\min}\left[\hat{\boldsymbol{Q}}(\hat{\theta})\right]$ is the eigenvector corresponding to the smallest eigenvalue of the matrix $\hat{\boldsymbol{Q}}(\hat{\theta})$ and the $det\ \left[\hat{\boldsymbol{Q}}(\theta)\right]$ is the determinant of the matrix $\hat{\boldsymbol{Q}}(\theta)$.

From (18)-(26), we observe that the DOA estimation and array shape calibration can be achieved simultaneously just using an one-dimensional search over FOV(Field of view) of array. Besides with uniform linear array, due to the Vandermonde structure of the ideal steering vector, a polynomial rooting with degree of 4 can also be utilized in light of the idea behinds ROOT-MUSIC algorithm.

Although the small position error assumption is not made in the above shape calibration algorithm, it is always assumed that the position uncertainties are not large to such an extent that the phase delay ambiguity identified in [11] is present.

Finally, it is obviously that the idea proposed here can easily be extended to array calibration in the presence of direction dependent gain and phase calibration.

IV. Simulation Results

In this section, simulation results are presented to illustrate the performance of the new algorithm. Simulations are carried out for a nominal uniformly linear array of 16 sensors with one half-wavelength inter-sensor spacing. Three instrumental sensors are added to form a nominal uniformly linear array of 19 sensors. The corresponding reference coordinate system, nominal array geometry and perturbed array geometry are shown in the fig.1. The actual sensor positions are arbitrarily fixed and allowed to randomly vary from the nominal sensor positions within the range $\pm 0.5\lambda$ in Y-coordinates and $\pm 0.25\lambda$ in X-coordinates. Two narrowband uncorrelated sources with equal power impinge on the array, from the far filed, at distinct directions 30° and 40° w.r.t the broadside of array. The SNR=20dB is defined as the ratio of each signal power to the noise power at each sensor. 200 snapshots are used to estimate the array covariance matrices. The number of sources is assumed known.

Fig.2 shows the spatial spectra obtained form the new algorithm (18). In table 1-2, we also demonstrate the estimated value and real value of the sensor positions.

From the results presented above and many other simulations with similarly favorable results, it can be concluded that the array shape calibration algorithm proposed here provides us an efficient and relatively inexpensive array shape calibration scheme.

V. Conclusion

In this paper, we propose an efficient array shape calibration algorithm by using two non-disjoint sources in unknown direction and three instrumental sensors. It can be applied to arbitrary array geometries including linear arrays. The new method is computationally abstractive and relatively inexpensive. Besides, small position error assumption is relaxed to meet the need of array shape calibration in the presence of large position errors. Without any modification, the idea proposed here can also be extended to the array calibration in the presence of direction dependent gain and phase uncertainties.

References

[1] A. J. Weiss and B. Friedlander, "Array shape calibration using sources in unknown locations-A maximum likelihood approach", *IEEE Trans ASSP* Vol.37 no.12, pp1958-1966, Dec. 1989.

[2] A.J. Weiss and B. Friedlander, "Array shape calibration using eigenstructure methods," *Signal Processing*, vol. 22, no.3, pp. 251-258, Elsevier Science Publishers, 1991.

[3] J. H. Hong, "Genetic approach to bearing estimation with sensor location uncertainties". *IEE Electronic letter* Vol.29 no.23, pp.2013-2014. Nov 1993.

[4] B. P. Flanagan and K. L. Bell, "Improved array self-calibration with large sensor position errors for closed space sources". *Proc. 2000 Sensor Array and Multichannel workshop*, Cambridge, MA, March 2000, pp.484-488.

[5] C. R. Wan, J. T. Goh and C. S. Chia,"Array calibration based on Inverse subspace fitting", *IEEE 2001Conference and Exhibition on OCEANS* Vol 4, pp.2468-2491.

[6] B.P.Ng, S.Wee "Array shape calibration using source in known locations"*Proc ICCS/ISITA'92 Sigapore* pp.836-840.

[7] B. C. Ng A. Nehorai "Optimum active array shape calibation" *Proc. 25th Asilomar Conference on signal,system and computers 1991* pp.893-897.

[8] Ming Zhang and Zhao-Da Zhu "Array shape calibation using sources in known directions". *Aerospace and electronics Conference NAECON 1993* pp.70-73.

[9] Y. Rockah and P. M. Schultheiss, "Array shape calibration using sources in unknown locations-Part I: Far field sources", *IEEE Trans ASSP* Vol.35, no.3 pp286-317 Mar 1987.

[10] Y. Rockah and P. M. Schultheiss, "Array shape calibration using sources in unknown locations-Part II: Near field sources and estimator implementation", *IEEE Trans ASSP* Vol.35, no.6, pp724-735, Mar 1987.

[11] J.T.-H.Lo and S. L. Marple, "Observability conditions for multiple signal directon finding and array sensor localization". *IEEE trans SP* Vol.40, no.11, pp.2641-265, Nov.1992

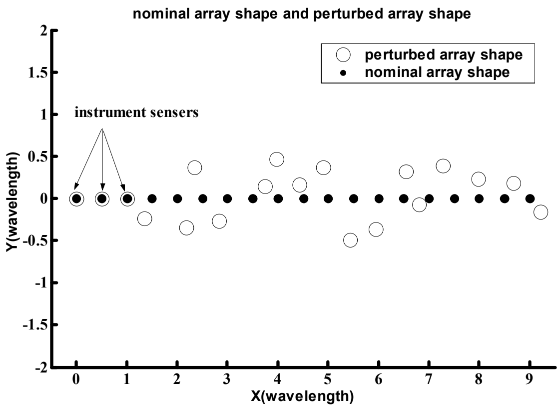

Fig. 1. Reference array coordinate system

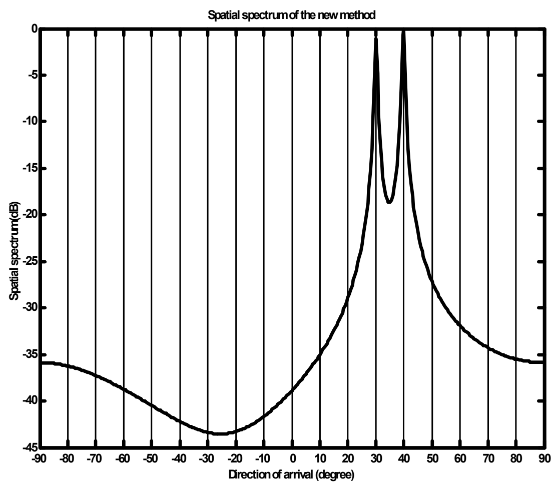

Fig. 2. Spatial spectrum acquired with new method

TABLE 1
ESTIMATED X-COORDINATES

Coordinates	Nominal	Actual	Estimated
X4	1.5	1.3454	1.3354
X5	2.0	2.1719	2.1783
X6	2.5	2.3370	2.3403
X7	3.0	2.8354	2.8450
X8	3.5	3.7471	3.7465
X9	4.0	3.9699	3.9840
X10	4.5	4.4200	4.4321
X11	5.0	4.9071	4.9029
X12	5.5	5.4325	5.4262
X13	6.0	5.9466	5.9390
X14	6.5	6.5458	6.5479
X15	7.0	6.8099	6.8093
X16	7.5	7.2691	7.2664
X17	8.0	7.9793	7.9755
X18	8.5	8.6849	8.6864
X19	9.0	9.2171	9.2150

TABLE 2
ESTIMATED Y-COORDINATES

Coordinates	Nominal	Actual	Estimated
Y4	0	-0.2356	-0.2257
Y5	0	-0.3397	-0.3426
Y6	0	0.3729	0.3721
Y7	0	-0.2621	-0.2687
Y8	0	0.1458	0.1447
Y9	0	0.4669	0.4560
Y10	0	0.1649	0.1547
Y11	0	0.3704	0.3711
Y12	0	-0.4901	-0.4884
Y13	0	-0.3630	-0.3602
Y14	0	0.3188	0.3151
Y15	0	-0.0698	-0.0706
Y16	0	0.3903	0.3921
Y17	0	0.2349	0.2389
Y18	0	0.1873	0.1888
Y19	0	-0.1539	-0.1521